New Design Ideas
Vol.2, No.2, 2018, pp.76-96

MATHEMATICAL EXPLANATION OF SPATIAL RELATIONALITY IN THE BUILT ENVIRONMENT

Harun Ekinoglu*

Istanbul Halic University, Department of Architecture, Istanbul, Turkey

Abstract. Cities change constantly in different size and portions. Conventional methods in GIS help to measure and explain spatial change through geographic causalities using geo-located vectorial data. This research focuses on the question of how to analytically measure the gradual effects of changing size, shape and scale factors upon the parts and whole interactions in the built environment. Proposed method in this paper introduces an alternative approach that translates vectorial data to scalar and measures built environment on a relational basis. The method developed for this question constructed upon Alexander's "Theory of Wholeness," Salingaros' "Inverse Power-Law Scaling," Tobler's "First Law of Geography" and Shannon's "Shannon's Information Entropy" theories. Through a dynamic grid interface, the proposed analysis tool translates various morphologic scenarios into scalar data and calculates Entropy-IQR (H) values specific for each grid-scale. For the analysis with various grid-scales, the cumulative of entropy-IQR values are inversely correlated to the degree of wholeness. Results obtained from the three groups of hypothetical case studies with slight and extreme differences prove the meaningful relationships between the spatial layouts and the entropy values created. The more extreme changes in the location and scale of the sub-constituents in the built system, the higher entropy that implies a lower degree of wholeness. For practitioners and local governments, the proposed method is allowing to run evidence-based measuring of wholeness in the built environment either to report or visualize the relative change and to test the possible design scenarios or analytically evaluate the after-effects of urban design projects in various scales before the implementation.

Keywords: *relationality, information entropy, entropy, complexity, wholeness.*

Corresponding Author: *Harun Ekinoglu, Assistant Professor, Istanbul Halic University, Department of Architecture, Istanbul, Turkey, e-mail: harun.ekinoglu@gmail.com*

Received: *17 October 2018;* ***Accepted:*** *16 November 2018;* ***Published:*** *03 December 2018*

1. Introduction

Tobler (1970), in his influential "First Law of Geography", states that everything is related to each other but near things are more related than distant things. Waldo Tobler's theory of geography seemed comprehensive and moderated and justified a number of assertions or theories on urban growth simulation (Miller, 2004). Miller states that, when it was first shared, the concepts of "nearness" and "relatedness" filled a gap needed in the core of spatial analysis and modeling. In the last decade, the rise of technology with geographic information systems established a ground in building a greater sophistication when measuring and analyzing various urban concepts (Miller, 2004). Miller (2004), also states that we do not need to conclude whether or not Tobler's First Law on nearness and relatedness justifies other empirical laws or descriptions or assertions on spatial formations. Since "nearness" and "relatedness" are space-related concepts they are related to diverse functionalities such as energy transfer or expenditure and so on. Miller in this sense notes that measuring the space through the concepts of near and related is not just to generate simple metric or geometric findings but to develop new spatial attributes that will allow measuring and modeling the geographic space through new complex interactions.

Correlation does not necessarily prove a causality. It can just imply an evidence that there is a relationship between two things. Two things with or without some other related variables can shape a certain causal relationship. The relationship or nearness and relatedness between a certain unit of the spatial formation to the rest of the spatial system requires a definitive method setting a certain way of data measuring and modeling. This method is expected to rationalize the process of how measured entities are considered. Such a method is expected to generate descriptive or confirmative findings that may help to explain a certain causality between two phenomena. This is a certain way of autocorrelation that measures the spatial interaction between selected spatial variables (Miller, 2004). Choice problems appear to be more meaningful when considering spatial proximities (Miller, 2004). Agents in this sense, through established rules that model the most possible behaviors, predicts the consecutive steps (Crooks *et al.*, 2007; Macal & North, 2005)**.**

Miller and Wentz (2003) suggest that near is central to spatial analysis and it's more flexible and powerful than commonly appreciated. Gatrell (1983) states that geographers do not have a commonly approved definition of space so that we can develop a mathematical definition by considering a set of objects and the relations between them. This kind of relations can be either qualitative or quantitative. Miller's comment for this is that geographers when thinking and discussing space, are using names or categories as meaningful phenomena on the land surface. The common knowledge an practice with distinguishing goe-spaces from other geo-spaces is the shortest-path relation between them. Shortest-path relation between geographic entities in mapping systems determine measurements of site-specific geospatial attributes. Shortest-path relation, by a straight line between two entities, defines a euclidian distance on the analytical plane. The nearness that TFL introduces and Miller discusses is not euclidian distance but relative and interactive nearness. This is a quasi-metric distance allows making alternative cartographic transformations and other visualization techniques possible.

Although, cyberspaces and the internet of things technology change and govern daily human life practices and shifts human-space interactions to a completely new level in the present time, one may question whether or not the concepts of relatedness and nearness of geographic space are significant like ever before. Places in the built environment are functionally getting nested and dissolved in and out of each other in cybernetics speed, spatial boundaries and time needed for real time-space interactions are no longer important. Both concepts beyond the shortest pathway of metric distance need to be explained by spatially continuous attributes that also explains a certain type of geographic interaction. In other words, in spatial terms, Miller's discussion on nearness reminds and helps developing new implications on understanding the concept of relatedness that emerges in space through the information of spatial probabilities that can be called and represented via entropic interactions way of relationality. Entropic interaction way of spatial relatedness is a form of agent-based modeling approach that employes Shannon's information entropy and models how spatial units of particular built areas co-create a gradual effect and deform an interface grid accordingly.

One of the other important analysis and modeling functions for measuring the spatial relationality is the Visibility Graph Analysis (VGA) as a 2D layout analysis in Space Syntax method. Braaksma and Cook (Turner, 2001) first developed VGA as an analysis of the built environments. The main purpose of the analysis is to calculate the co-visibility of various units through an adjacency matrix in the representation of the relationships (Turner, 2001). Based on quantifying the existing visibility relationships, the analysis tool "Depthmap" calculates and visualizes how satisfyingly visible a space based on its function. Later on, Turner and Penn (1999), parallel to the developments in space syntax method, proposed a comprehensive visibility analysis method considering of several graph metrics and measures that are being benefitted in Space Syntax. Briefly, the purpose of the VGA analysis is to generate a clustering coefficient. Clustering coefficient is the ratio of connected vertices to the number of vertices that are possibly connected. Visibility Graph Analysis, in its own terms, reveals the visible relationality of a built system embedded in its configuration and the degree of wholeness emerges through it.

According to Jiang (2017), one of the scholars who densely study on the wholeness of complex space systems, the wholeness emerges through a life-giving order or a living structure. Although not similar to the method and the terms presented in this paper, he combines diverse methods from space syntax to various topology-oriented geographic representations that view and measure the city and parts of the city as a whole. He measures the points, lines, and polygons, and in particular how these geometric primitives constitute living structures.

Salingaros and West (1999) state that we cannot expect to describe a complex system by a field equation because the complex systems are correlational rather than casual. There is a relationship between concurrence and scaling. Hence, to grasp the complex systems the best description is probability distributions (Salingaros & West, 1999). To measure this kind of complexity, every probability that is possible in a multiscalar multiplicity matters since information entropy measures the uncertainty arises through probability distribution. Correlation between the probability and entropy is parabolic and is further explained in the following part.

One of the biggest concerns and motivation behind this research is the remarkable loss of scaling hierarchy that is first defined as "inverse-power scaling" by Salingaros and West (1999), in cities due to contemporary architectural practice and fashion fueled by massive decontextualized reproductions on urban land. Wholeness implies smaller elements and thus smaller scales than the large ones (Salingaros, 1997). This is also true for the scales. Change in size, form, and shape of the subunits in the built environment affects morphologic concurrence and adaptations and results as spatial incompatibility, disorganized complexity (Salingaros, 2018), and mental and physical discomfort for the users.

2. Method and Data: Why and how to use Shannon Information Entropy

Entropy concept was first used as a term of thermodynamic systems in the nineteenth century. The second rule of thermodynamics says that every living or nonliving system has an amount of free energy, and it always moves towards equilibrium (Bailey, 2015) which the entropy increases to realize. In other words, a system spontaneously evolves towards a less ordered state. Nature tends to disorderliness more than order. The probability of a disordered or irregular occasion is higher than ordered and regular one (Shannon, 1948). There is an act of seeking equilibrium, and maximum entropy is thus what leads to disorderliness. Since the probability of a disordered state is higher than an ordered state, entropy always increases but never decreases. Maximum entropy takes a system to the collapse and death. Briefly, energy or substance in nature cannot be vanished but evolve from one state to another. Entropy is the measure of this evolution or transformation as graphically illustrated in Figure 1.

Figure 1. From the order to disorderliness: a graphic definition of entropy

In addition to thermodynamic entropy, information entropy was first introduced by German mathematician Claude Shannon (1948) as a basic concept in information theory measuring the average missing information on a random source (Jat *et al*., 2007). Shannon's entropy, originated from information theory, is a measure of uncertainty of conveyed information over a noisy channel (Bailey, 2015; Jat *et al*., 2007). The larger the value of Shannon's entropy, the higher is the uncertainty of information conveyed. Shannon developed the mathematical explanation of the information theory and focused on how to minimize the loss of information in revealing a message on another point. Entropy (H), in this sense, is a measure of information. "H" is dependent on the number of information categories, K. Higher the number of categories conveyed by a particular information, less probability of the same type of categories to gather. High entropy is the most probable, and yet the least predictable state that leads to disorder (Versoza & Gonzales, 2010; Bailey, 2015). Hence, in such case, the entropy is always towards most probable or most likely state. When the entropy is the highest, the data categories embedded in the information get to the most random state where the most uncertainty occurs.

The same approach is also valid for the built environment. Thus, Shannon's entropy is convenient in measuring the uncertainty of morphologic occurrences in urban settings in various scale levels. The use of Shannon's entropy in this research is expected to provide insights into the notions of randomness, typicality, and disorder about the hidden codes of the morphologic occurrences in cities. In the built context, employing the entropy concept is expected to find out the state of randomness and disorderliness nested in the relative distribution of built elements of all kind.

The method, using Shannon's entropy, helps to generate values for each grid unit as spatial attributes. Measuring the built clusters and comparing one to another in changing scales is the basis of the proposed method in understanding the interplay between scale and changing level of spatial uncertainty. Shannon's entropy in this respect has a critical role in translating and reproducing the data and providing a new insight or tendency towards disorderliness, about the nature of the analyzed morphologic co-occurrence. According to Leibovici (2009), use of Shannon's entropy on the bare distribution of a particular number of data categories with different configurations does not help to describe the entropy of each configuration. Scholars in this respect suggest integrating some specific spatial aspects or control definitions into the entropy calculation (Leibovici, 2009).

Shannon's entropy was derived from information equation (Wang, 2016).

Information equation is formulated as:

$$I(p) = -log_b P,$$

P is the probability of the event happening;
b is the base and the unit of measurement generated by the base 2 in information theory is bits.

An exemplary calculation of the information by tossing a coin:

There are two probabilities for a coin and they are 0,5 head and 0,5 tail. If we toss the coin and get either head or the tail, we get 1 bit of information as in the below equation and graph appears; $I(head) = -log_b(0.5) = 1bit$

Maximum entropy is achieved when all probabilities are equally likely and there is no ability to guess. It is, in other words, the case of maximum surprise. In the case of a coin, the probability is 0,5 both for head and tail. It makes the uncertainty and thus the entropy maximum that is 1bit as appears in the graph in Figure 2. Minimum entropy occurs when one symbol is certain and others are impossible as in the graph. In other words, when there is no surprise, there is no uncertainty.

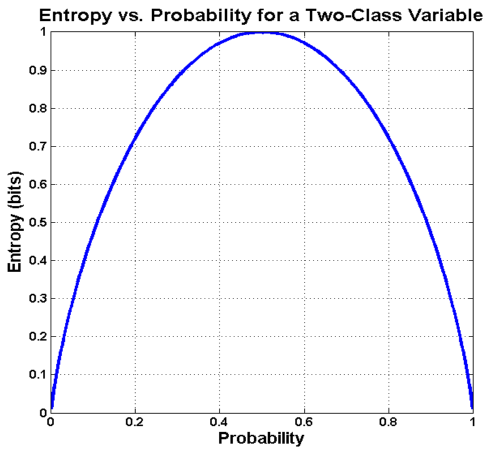

Figure 2. Entropy vs. Probability for a two-class variable in case of a coin toss

In physics, the word entropy has important physical implications as the amount of "disorder" of a system. In mathematics, a more abstract definition is used. The (Shannon) entropy of a variable X is defined as bits, where $P(x)$ is the probability that X is in the state x, and $Plog_2 P$ is defined as 0 if $P = 0$. The joint entropy of variables $X_1, \ldots \ldots \ldots X_n$ is then defined by

$$H(X) = -\sum_x P(x) log_2[P(x)],$$

$$H(X_1, \ldots, X_n) = -\sum_{x_1} \ldots \sum_{x_n} P(x_1, \ldots, x_n) log_2[P(x_1, \ldots, x_n)].$$

"G" as unit-based built probability and Shannon's Information Entropy (H) by definition are two co-dependent spatial terms in the proposed method. G is termed to be the measure of scale-based built density for each unit as illustrated in Figure 3 and formulated in the below equations. Entropy (H), on the other hand, is the measure of uncertainty that each grid unit holds considering the connected units as illustrated in Figure 3 lower row G for a built context implies scale responsiveness and is generated for every particular grid unit area while H is generated for any unit area that is a neighbor to eight connected equivalent units. It is simply the entropy emerged through the relative interaction among a particular unit area and its adjacent neighbors and is formulated in the equations (1), (2), and (3) in below.

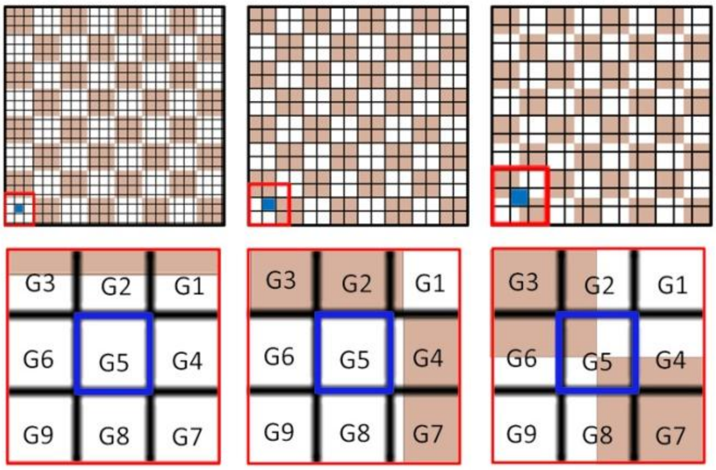

Figure 3. Unit positions and their interactions with adjacent units in the calculation of *G*, *P* and *H* values

Referring to Shannon's entropy as formulated in below equations (1), (2), and (3), given i is a central unit adjacent to 8 equivalent units, n is the 9 adjacent units that form square sub-regions throughout the grid system. G_i, for the i^{th} unit of the n units of a sub-region in a grid system, is the unit-specific built portion. P_i is the built portion by the unit i divided by the total built portion of the nine units sub-region where i is the central unit. For instance, as in Figure 3, the entropy (H) for the 9-units sub-area where $G5$ it the central unit with $G5$ value. P_i is that $G5$ is divided by $G1,\ G2,\ G3,\ G4,\ G5,\ G6,\ G7,\ G8$ and $G9$ values. P_i multiplied by $logb(1/Pi)$ where the base for the logarithm is 2. H_i is calculated by taking the sum of each of the nine P_i values in the 9-units system.
Maximum adjacency in Figures 3 and 4 refers to the rule that for each unit that is adjacent to 8 connected units can only be calculated P. In other words, the units by the grid edges are exempted in entropy calculation as illustrated in Figure 4 since they do not meet this criterion.

$$G_i = \frac{Built\ portion\ of\ pixel\ i}{Total\ built\ pixel\ area} \tag{1}$$

$$P_i = G_i / \sum_i^n G_i \tag{2}$$

$$H_i = \sum P_i \cdot log\left(\frac{1}{P_i}\right) \tag{3}$$

Each grid unit in Figure 3 matches a particular space and thus a particular portion of a morphologic occurrence represented by a G value. If a unit partially frames a building or a group of buildings, the algorithm as in equation 1, assigns a ratio for the area of the built part divided by the total unit area depending on the scale of the grid. It assigns $G = 0$ when the unit area is totally no-building, and $G = 1$ when it fully frames a building or group of buildings.

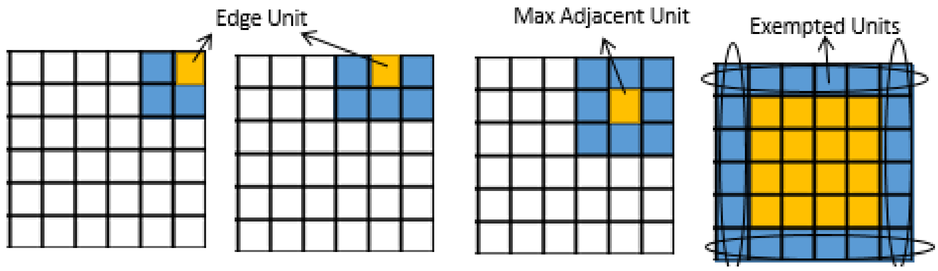

Figure 4. Exempted units since they do not meet the maximum adjacency rule in calculating the *P* values

IQR (Interquartile Range) is a statistical data measuring method that does a discretization for the data with different spreads. It arranges the values from the smallest to the biggest. For discretization of the deviations along the data, IQR plays a role in extracting the “middle fifty” where it draws a specified data as graphed using the Box and Whisker Plot in Figure 5. The extremes of the data eliminated, and it is where the bulk, middle fifty, of the data falls in. It is preferred over many other measures of spread in statistics when reporting about multivariate data sets. Because each output is scale-dependent, the ranges of the quartiles change as the scale of the analysis changes. In other words, each IQR for a specific scale relies on the changing morphologic states that are framed by different size of grid units. By the IQR method, each output dataset is reduced to a single value. Multiple analyses for varying scales allow generating multiple IQRs.

The total sum of Entropy-IQRs for various scale levels gives the outcome of the study. Knowing the values of quartiles, $Q1$ and $Q3$, has critical importance. The position of the quartiles, between 0, 00 and 1, 00, can be highly distinct, somehow close to each other or juxtaposed which at the end explains how the IQR is created in fact. When the quartiles are located distinctly far from each other, IQR gets a higher value. This also points to the deviations that exist along the entropy dataset. The remarkable deviations in the dataset point to the remarkable differences among the morphologic formations framed by the grid units, in other words among the G values, unit based built probability. The differences gradually affect the P as mentioned in above equation (2), the relational probability of total nine adjacent units as seen in Figure 3, and thus the entropy (H) value in equation (3) explained above.

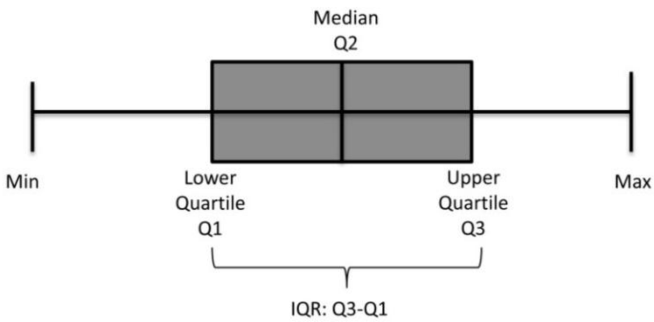

Figure 5. Box and Whisker Plot. Generating IQR statistical value for a multivariate dataset

3. Entropic Interaction among the units of a grid system superimposed a built area

Entropic interaction is a state of tension might evolve into a process of adaptation towards a new complexity which in nature leads to uncertainty since the energy of things cannot be captured forever and constantly transferred. Salingaros (2018) states; adaptation generates useful complexity through a feedback among diverse complexities. These complexities can be created between imposed & self-generated complexities or imposed & imposed ones. Cities, one of the biggest inventions of human-being, are in constant change in several size and scales so that the adaptation that cities experience is a good example of this phenomena. Salingaros (2018) however notes that building the wrong type of complexity results in dysfunctional buildings that leads to waste of enormous energy resources to maintain. This is also quite relevant with the geographic and spatial relatedness that Tobler draws in "*first law of geography*." What is more related will naturally adapt easier and faster using optimal energy that leads to maximum efficiency. Information entropy theory in this research is employed to unearth the measure or degree of a tendency towards uncertainty through this adaptation.

Entropic Interaction, in addition to the multi-scalar analysis, is the second data-visualization function of the tool. Principally entropy indicates a level of the tendency towards uncertainty or disorderliness. Higher entropy means a higher tendency towards uncertainty. From the spatial relatedness point of view, a particular built area framed by a grid unit is spatially and geographically related with its adjacent units and intrinsically through each other's entropic states. In other words, there is constant interaction among the system parts. This can be visualized as an entropic interaction modeling approach.

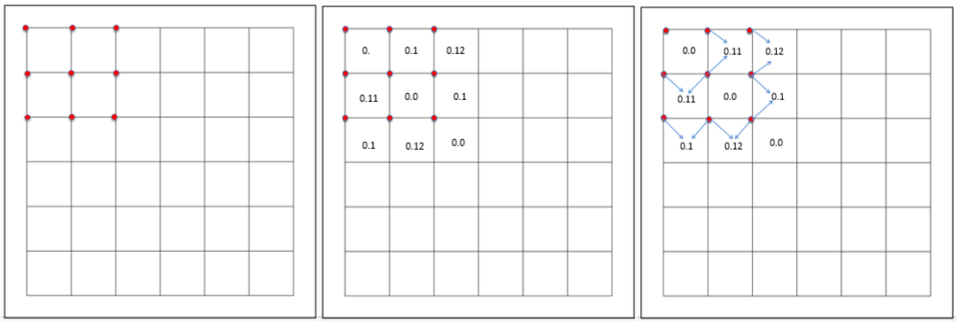

Figure 6. Adjacent grid units with entropy values and the way they create a joint force

Each grid system highlights the vertices of each unit area that they intersect. Mutual effect interaction, described by Brannon (2008), is here interpreted as the displacement (deformation) of the four vertices of each unit due to the forces generated by the entropy values as explained in as in Figures 6 and 7. Mutual effect deformation, in entropic interaction, is interpreted as the displacement of the four vertices of each grid unit due to the joint forces generated by different entropic zones aligned by each vertex. The displacement of the vertices is due to the joint strength of the entropic intensities belonging to the connected units. Simply the higher entropy pulls stronger and thus deforms more. By using the Centripetal Catmull-Rom curves algorithm (Yuksel *et al.*, 2011), all the vertices are then reconnected via first by straight lines and then curves as in Figure 7. There are also other widely used interpolation curves with the same properties (NURBS, B-Splines, etc).

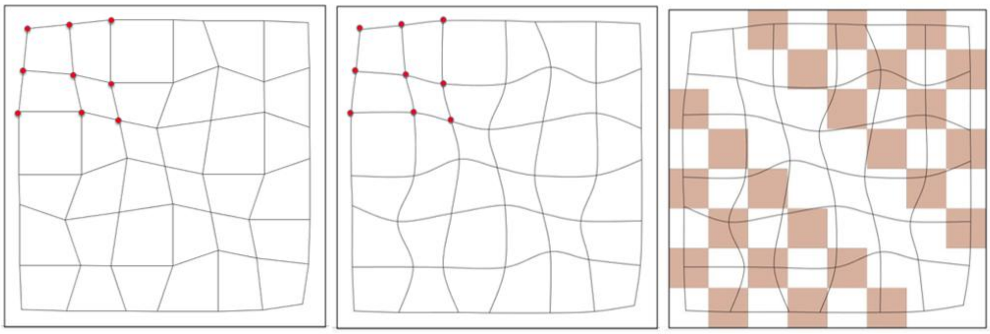

Figure 7. Displacement of the vertices and reconnection via straight lines and straight lines converted to the curves

4. Hypothetical Case Studies

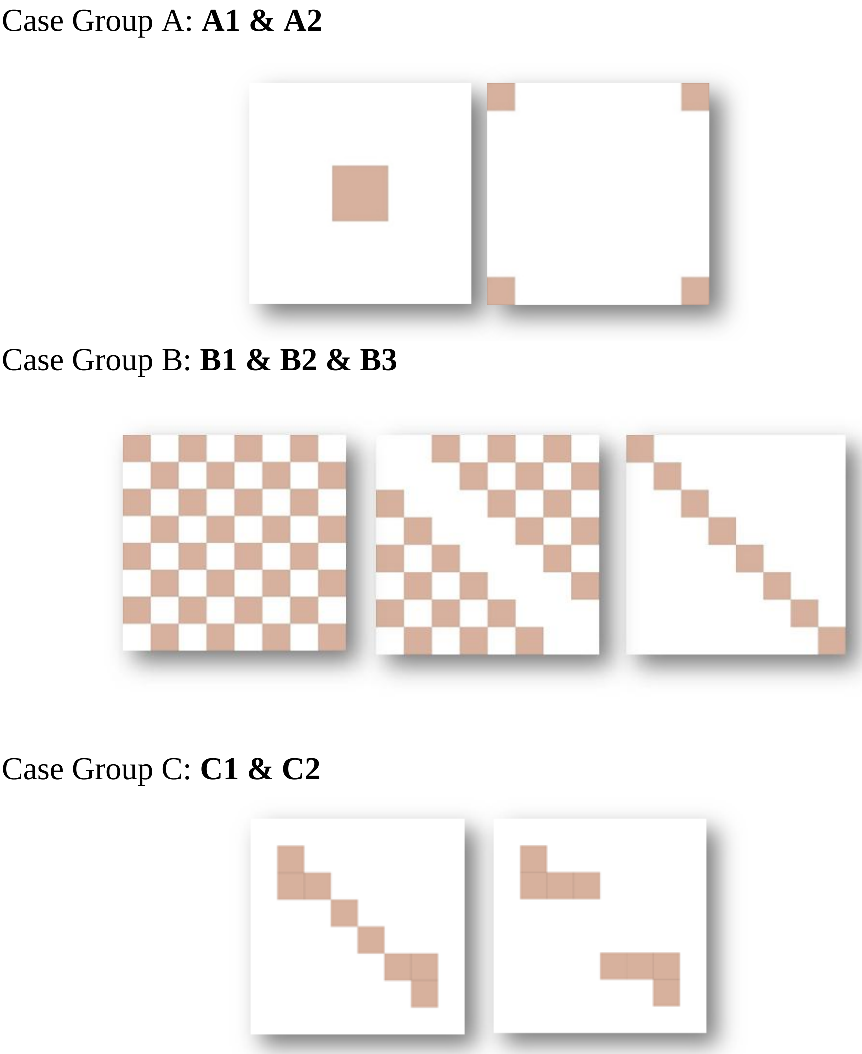

Figure 8. Three groups of hypothetical case study morphologic formations

Three groups of hypothetical case study configurations with slight or excessive differences have been prepared to analyze and see what kind of entropic changes are triggered by what sort of configurational changes in the order of the pattern or morphologic layout in each case. The particular hypothetical cases might show excessively transformed versions of different configurations. Equivalent units through changing grids of different scales frame varying morphologic formations. Some morphologic formations are purposefully as simple as to be able to explain the cause-effect relationships. All of the analyses have been done using the same grid scales with particular pixels. In doing so, each hypothetical configuration will allow grasping the order and entropic relationships from Shannon's Entropy point of view. The analyses, besides generating MSA (multi-scalar analysis) graphs and MSA Data Tables, also helped to generate 1) unit-specific morphologic probability (G) visualizations, and 2) Entropic interaction (EI) visualizations. In each visualization, we are able to see the relative effect of micro-spatial changes upon the entire formation. Small or large, every single change creates a new morpho-information. However, it is a change in the vectorial data. A vectorial data is the type of data that we know the specific location in the real-world system. It might be represented using points, lines and areas. The term scalar is a linear algebra term. It helps to distinguish a single number from another scalar data no matter in what purpose it is used. The vectorial nature of spatial relatedness, using the method in this study, is converted to the scalar data using Shannon's entropy.

Each of the seven hypothetical case studies, in Figure 8, have legible and definite layouts. Not to get confused in understanding the morphologic-change & entropic outcome cause-effect relationship, each has been designed purposely, step by step, to see what happens in entropic data answering to the fundamental morphologic changes. To achieve this, the same grid-scale levels have been used for all of the case study analyses. Grid scales that have been used are: 100 : 15px.15px; 200:30px.30px; 300:45px.45px; 400:60px.60px; 500:75px.75px; 600:90px.90px; 700:105px.105px; 800:120px.120px; 900:135px.135px; 1000:150px.150px. For all the case studies, a same single grid-scale level could have been used. However, that would not allow monitoring the change in the entropic state of the overall layout answering to the grid-units from small to the large sizes.

Hypothetical Cases A1 & A2: 2 Different 1024 x 1024 pixels case areas

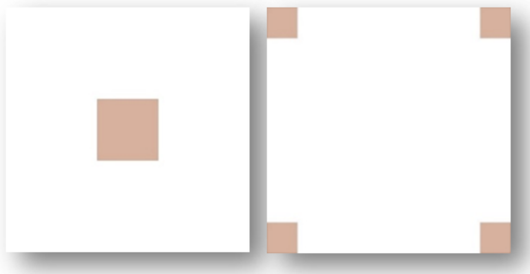

Figure 9. Hypothetical cases A1 & A2

Configurations	: A1: 4 equivalent units squared by the center, A2: 4 equivalent units individually located by the corners of a square
ΣH-IQRs	: A1:0,1bit & A2:0,0bit

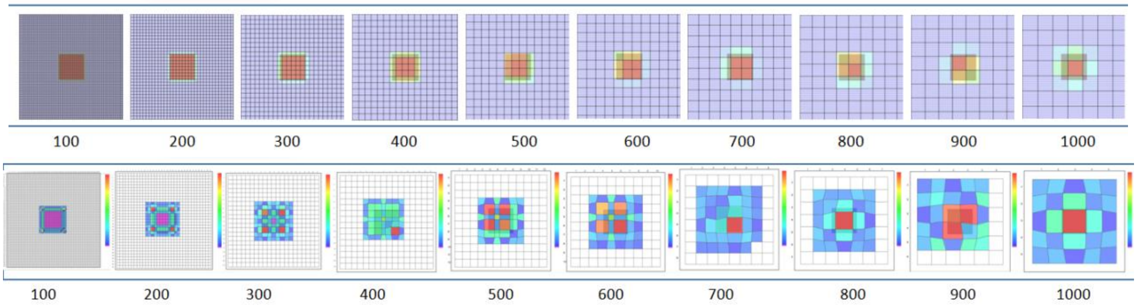

Figure 10. Analyses for the grids of varying scale levels superimposed upon the raw data A1 case study area and G (Upper) and EI (Lower) visualizations

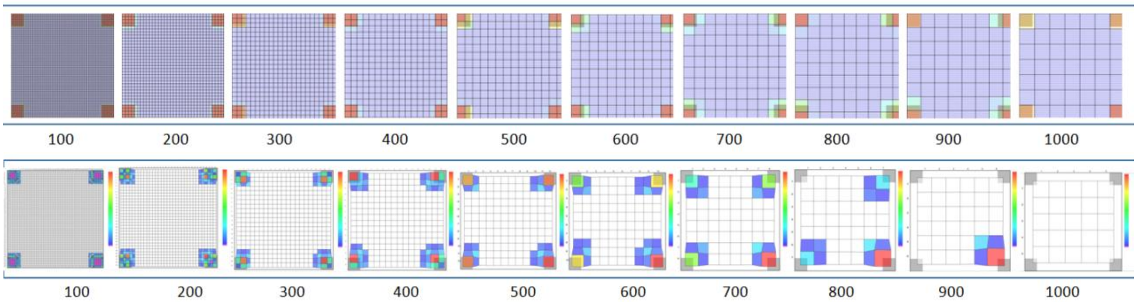

Figure 11. Analyses for the grids of varying scale levels superimposed upon the raw data A2 case study area and G (Upper) and EI (Lower) visualizations

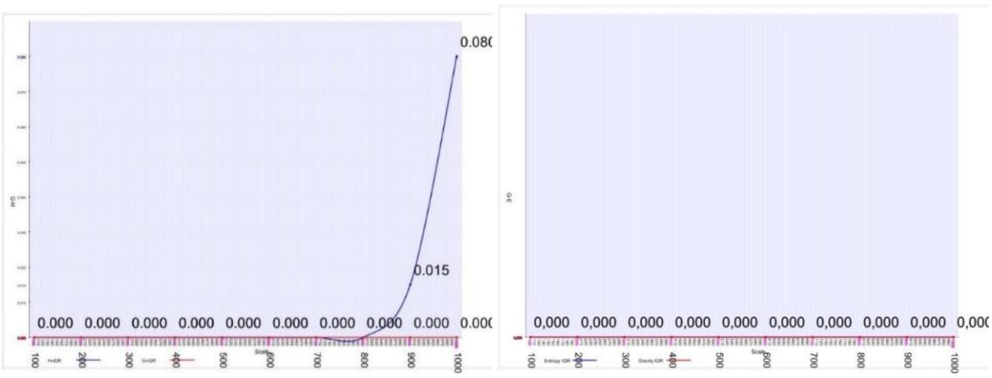

Figure 12. MSA graphs for the hypothetical cases A1 & A2

Table 1. MSA Data Table for the hypothetical case A1

Grid Scale Level	PA (Pixel Area)	Total Cell	NA Cell (exempted)	%	G-IQR	G-Q1	G-Q3	H-IQR	H-Q1	H-Q3
100	15px.15px	4761	272	5.7	0,000	0,000	0,000	0,000	0,000	0,000
200	30px.30px	1225	136	11.1	0,000	0,000	0,000	0,000	0,000	0,000
300	45px.45px	529	88	16.6	0,000	0,000	0,000	0,000	0,000	0,000
400	60px.60px	324	68	20.9	0,000	0,000	0,000	0,000	0,000	0,000
500	75px.75px	196	52	26.5	0,000	0,000	0,000	0,000	0,000	0,000
600	90px.90px	144	44	30.5	0,000	0,000	0,000	0,000	0,000	0,000
700	105px.105px	100	36	36.0	0,000	0,000	0,000	0,000	0,000	0,000
800	120px.120px	81	32	39.5	0,000	0,000	0,000	0,000	0,000	0,000
900	135px.135px	64	28	43.7	0,000	0,000	0,000	0,020	0,000	0,150
1000	150px.150px	49	24	48,9	0,000	0,000	0,000	0,080	0,000	0,080

Table 2. MSA Data Table for the hypothetical case A2

Grid Scale Level	PA (Pixel Area)	Total Cell	NA Cell (exempted)	%	G-IQR	G-Q1	G-Q3	H-IQR	H-Q1	H-Q3
100	15px.15px	4761	272	5.7	0,000	0,000	0,000	0,000	0,000	0,000
200	30px.30px	1225	136	11.1	0,000	0,000	0,000	0,000	0,000	0,000
300	45px.45px	529	88	16.6	0,000	0,000	0,000	0,000	0,000	0,000
400	60px.60px	324	68	20.9	0,000	0,000	0,000	0,000	0,000	0,000
500	75px.75px	196	52	26.5	0,000	0,000	0,000	0,000	0,000	0,000
600	90px.90px	144	44	30.5	0,000	0,000	0,000	0,000	0,000	0,000
700	105px.105px	100	36	36.0	0,000	0,000	0,000	0,000	0,000	0,000
800	120px.120px	81	32	39.5	0,000	0,000	0,000	0,000	0,000	0,000
900	135px.135px	64	28	43.7	0,000	0,000	0,000	0,000	0,000	0,000
1000	150px.150px	49	24	48,9	0,000	0,000	0,000	0,000	0,000	0,000

The A1 hypothetical case has been designed as 4 equivalent units squared by the center of the area forming a solid square geometrically situated exactly by the center. The super-block, that acts a single built entity, is an enclosed form that never interacts with the surrounding large open space. The form, no doubt, defines a monolithic and introvert solid. The analysis results, in Table 1, indicate that the quartiles, H-Q1 and H-Q3, that define the H-IQR start to affect the quartiles in 900 and 1000 grid-scale levels. In other words, the H-IQR values start to be higher than 0,00 from 900. This is because the total size of open spaces compared to the total size of the identical blocks is far larger in general so that the G values in the maximum adjacent system (nine-units system) start to create H values as the grid unit sizes get larger. The third quartiles of the H datasets generated for 900 and 1000 grid scales become higher than 0,00. Logically the outcomes of the analysis indicate that the degree or intensity of spatial change is about the scale level we examine the relationality of morpho-information that occurs in a particular grid scale level. ΣH-IQR is 0,1bit.

The A2 hypothetical case has been designed as four equivalent units individually located by the corners of the case area forming a symmetrically perfect square in the equal distance to the center. In contrary to the previous hypothetical case of A1, the A2 case study area defines a larger possibility of interactions among the built and non-built areas. The layout is monolithic and allows the user to examine what happens when a super-block gets equally dismantled into four identical sub-blocks and each locates in the same distance to the center. Similar to the previous case of A1, there is still a large open space in the system what shapes the quartiles, as seen in Table 2, for all the grid-scale levels. In all grid-scale levels, both the G and H quartiles and thus the IQRs come out as 0,00. This is obviously due to one reason: the built entities by the corners, in any grid-scale level, do not get the ability to affect first and third quartiles (Q3 and Q1). The open space keeps creating the Q1 and Q3 values for all the grid-scale levels as 0,00 meaning non-built nine-units system where there is no built entity to trigger an entropic state that dominates the G and thus H datasets as seen in Figure 11. When comparing the case A1 with the A2, one can say that not only the way a built entity is clustered but also the location of the spatial entities entropically matters. This outcome is supported by Leibovici's suggestion of generating entropy for spatial distributions requires considering the adjacencies as a proximity and relationality factor. ΣH-IQR for the A1 is 0,0bit meaning there is no entropy in the system. As a result, the cumulative H-IQR is lower for the case A1 when compared to the case A2, while in the case A1, the monoblock, affects a greater area in entropic interaction while the equally dismantled sub-blocks of A2 by its equivalent corners until 900 scale.

Hypothetical Cases B1 & B2 & B3 : 3 Different 1024 x 1024 pixels case areas

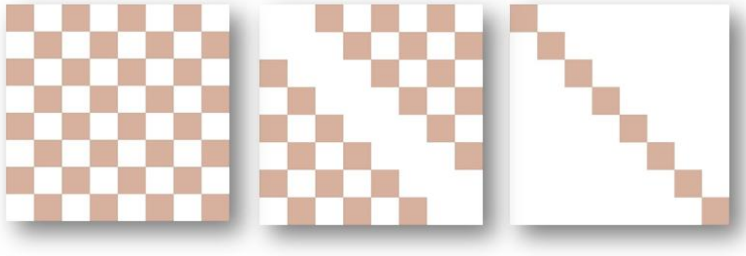

Figure 13. Hypothetical cases B1 & B2 & B3

Configurations : B1: Full Chessboard pattern of equivalent pieces; B2: Chessboard pattern with eight diagonally missing equivalent pieces B3: Eight equivalent pieces diagonally located from upper left to lower right corners

ΣH-IQRs : 0,64bit; 0,9bit; 0,57bit.

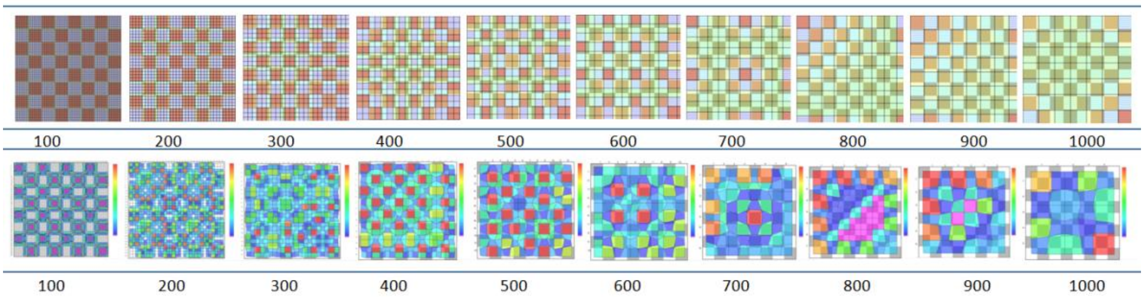

Figure 14. Analyses for the grids of varying scale levels superimposed upon the raw data B1 and G (Upper) and EI (Lower) visualizations

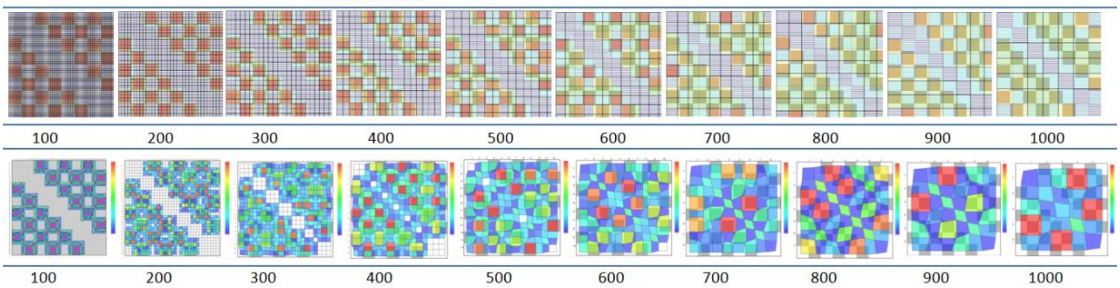

Figure 15. Analyses for the grids of varying scale levels superimposed upon the raw data B2 and G (Upper) and EI (Lower) visualizations

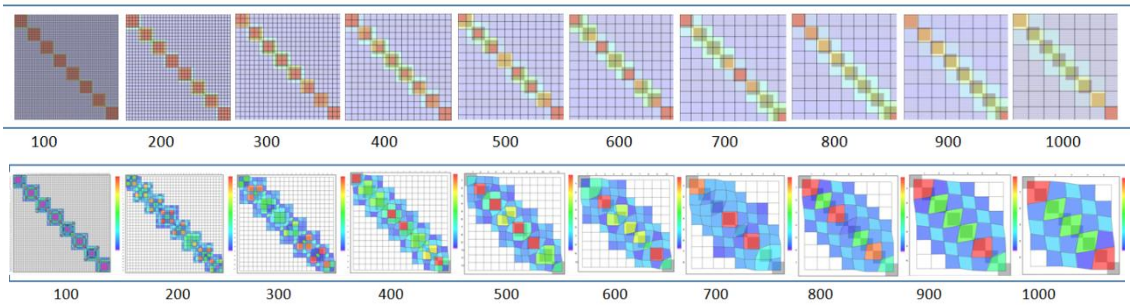

Figure 16. Analyses for the grids of varying scale levels superimposed upon the raw data B3 and G (Upper) and EI (Lower) visualizations

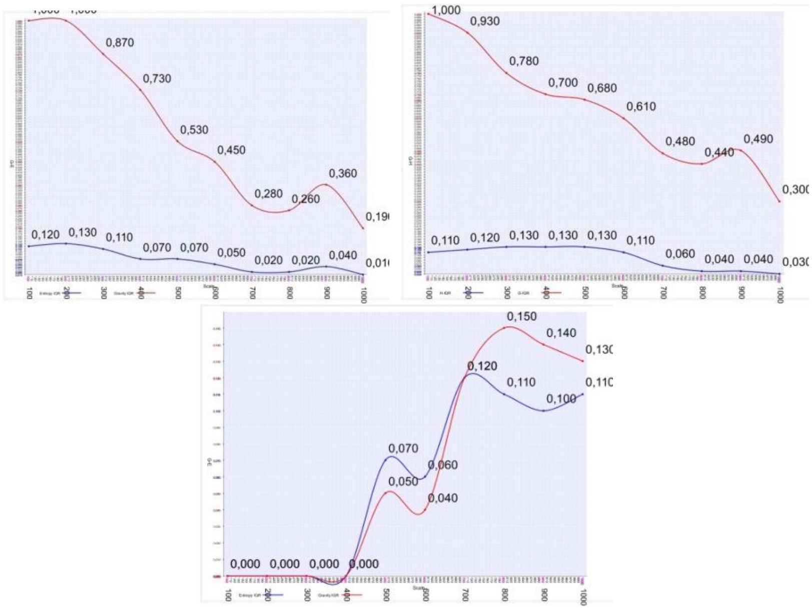

Figure 17. MSA graphs for the hypothetical cases B1, B2, and B3 case studies

Table 3. MSA Data Table for the hypothetical case B1

Grid Scale Level	PA (Pixel Area)	Total Cell	NA Cell (exempted)	%	G-IQR	G-Q1	G-Q3	H-IQR	H-Q1	H-Q3
100	15px.15px	4761	272	5.7	1	0	1	0.12	0	0.12
200	30px.30px	1225	136	11.1	1	0	1	0.13	0	0.13
300	45px.45px	529	88	16.6	0.87	0.06	0.93	0.11	0.03	0.14
400	60px.60px	324	68	20.9	0.73	0.13	0.86	0.07	0.07	0.14
500	75px.75px	196	52	26.5	0.53	0.24	0.77	0.07	0.075	0.14
600	90px.90px	144	44	30.5	0.45	0.26	0.705	0.05	0.08	0.13
700	105px.105px	100	36	36.0	0.28	0.37	0.65	0.02	0.1	0.12
800	120px.120px	81	32	39.5	0.26	0.365	0.62	0.02	0.1	0.12
900	135px.135px	64	28	43.7	0.36	0.315	0.675	0.04	0.09	0.13
1000	150px.150px	49	24	48,9	0.19	0.38	0.57	0.01	0.11	0.12

Table 4. MSA Data Table for the hypothetical case B2

Grid Scale Level	PA (Pixel Area)	Total Cell	NA Cell (exempted)	%	G-IQR	G-Q1	G-Q3	H-IQR	H-Q1	H-Q3
100	15px.15px	4761	272	5.7	1	0	1	0.11	0	0.11
200	30px.30px	1225	136	11.1	0.93	0	0.93	0.12	0	0.12
300	45px.45px	529	88	16.6	0.78	0	0.78	0.13	0	0.13
400	60px.60px	324	68	20.9	0.7	0	0.7	0.13	0	0.13
500	75px.75px	196	52	26.5	0.68	0	0.68	0.13	0	0.13
600	90px.90px	144	44	30.5	0.61	0	0.61	0.11	0.02	0.13
700	105px.105px	100	36	36.0	0.48	0.08	0.56	0.06	0.065	0.125
800	120px.120px	81	32	39.5	0.44	0.13	0.57	0.04	0.08	0.12
900	135px.135px	64	28	43.7	0.49	0.14	0.63	0.04	0.09	0.13
1000	150px.150px	49	24	48,9	0.3	0.24	0.54	0.03	0.09	0.12

Table 5. MSA Data Table for the hypothetical case B3

Grid Scale Level	PA (Pixel Area)	Total Cell	NA Cell (exempted)	%	G-IQR	G-Q1	G-Q3	H-IQR	H-Q1	H-Q3
100	15px.15px	4761	272	5.7	0	0	0	0	0	0
200	30px.30px	1225	136	11.1	0	0	0	0	0	0
300	45px.45px	529	88	16.6	0	0	0	0	0	0
400	60px.60px	324	68	20.9	0	0	0	0	0	0
500	75px.75px	196	52	26.5	0.05	0	0.05	0.07	0	0.065
600	90px.90px	144	44	30.5	0.04	0	0.04	0.06	0	0.06
700	105px.105px	100	36	36.0	0.12	0	0.12	0.12	0	0.12
800	120px.120px	81	32	39.5	0.15	0	0.145	0.11	0	0.11
900	135px.135px	64	28	43.7	0.14	0	0.135	0.1	0	0.1
1000	150px.150px	49	24	48,9	0.13	0	0.125	0.11	0	0.105

Case B1 is a perfect chessboard pattern with no missing pieces and distortion. As the grid unit size increases, the co-effect of the adjacent units gets smaller and so the relational effect of the adjacent units does. In such a monotonous context of identical massive sub-constituents, the system reveals approximate entropic states (H-IQR) in each scale level due to recurring scaling hierarchy through the fractal geometry in every scale level. One can conclude that such kind of order implies stabile or approximate entropic situations since the unit-specific G values stay same or approximate. Entropic stability might be a positive state, yet it is the topic of another discussion since massive multi-scalar and fractal may not always guarantee wholeness in spatial settings. Alexander himself also confirms this remark in the "*Nature of Order*" series. The cumulative H-IQR of this system is 0,64bit.

The hypothetical case B2 is another chessboard pattern with eight missing pieces in a crosswise way from southeast to northwest. The in-between space, between two identical clusters, remind a definitive torn or a border that disconnects every possible spatial interaction while enables a wide range of potential interventions. This situation, by all means, creates uncertainty about the design process and more specifically draws a state of openness to every possible spatial intervention. The definition of information entropy says that the higher number of data types in a particular information, the lower the probability of the same type of data to get together. In spatial terms, the higher the morphologic ability, the more uncertainty triggered in space. This analogy is being supported by the high cumulative entropy of 0,9bit, that the system reveals. Looking at

Table 4, the analysis results indicate that as the grid scales get larger, so does the units and the entropic state emerges through the interaction of the adjacent units that getting insignificant and decreasing the H-IQR values. Entropic Interaction, in accordance with the morphologic formation of the case, gets shaped creating highly deformed units overlaid upon the spaces that bear scale jumps between built and unbuilt. That means the more atypical and undefined spaces in a built area, the larger the uncertainty and the bigger risk of spatial interventions.

Case B3 is a layout formed by eight equivalent spatial entities diagonally located from the upper left to the lower right corners. Each of the eight structural entities interacts with surrounding open space at maximum: each through four edges and thirty-two edges in total, in all system. This fact obviously brings, no matter what the grid scale is, a great deal of morphologic interplay, scale jumps and thus a larger interaction potential regarding the scale of the grid used for the analysis. The smaller the unit size, the lower the G and thus H quartiles, as seen in above Table 5, and the larger the unit size, the larger the G and thus H values and thus growing G-Q3 and H-Q3 quartiles' values. This is because of until the 500 grid scale, the open space affects both first and third quartiles. After 500 grid-scale level, even if not the first quartiles, the third quartiles are affected by the growing units sizes and they start to affect both third quartiles and thus the H-IQR. In the case study B3, there is a visible balance between G-IQRs and H-IQRs. In other words, the rise of "quartile-able" built formation creates and alters the entropic state. For this hypothetical case, one can say that the higher possibility of solid-void interactions, the more volatile G, and H states. This finding proves that entropic state is about the level of analysis. Different deformations in the EI visualizations for different grid-scale levels prove this finding. Entropic Interaction, in accordance with the morphologic formation of the case, the EI gets shaped creating highly deformed units overlaid upon the empty spaces. That means the more undefined spaces in a built area, the larger the uncertainty, the bigger risk of spatial interventions as seen in Figure 17. The cumulative H-IQR for the Case B3 is 0,57bit.

Hypothetical Cases C1 & C2: 2 Different 1024 x 1024 pixels case areas

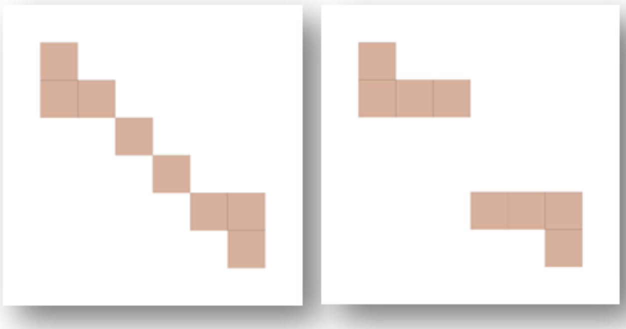

Figure 18. Hypothetical cases C1 & C2

Configurations : C1: Eight equivalent units with two pairs of identical and symmetrical modules, C2: Eight equivalent units grouped in total two identical and symmetrical modules

ΣH-IQRs : 0,60bit, 0,67bit

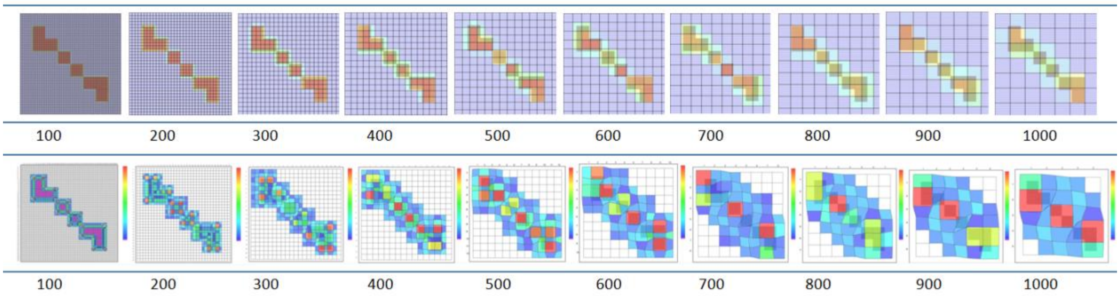

Figure 19. Analyses for the grids of varying scale levels superimposed upon the raw data C1 and G (Upper) and EI (Lower) visualizations

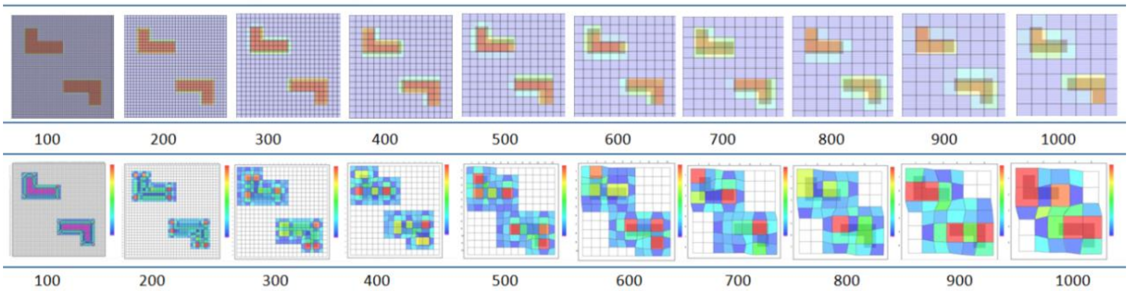

Figure 20. Analyses for the grids of varying scale levels superimposed upon the raw data C2 and G (Upper) and EI (Lower) visualizations

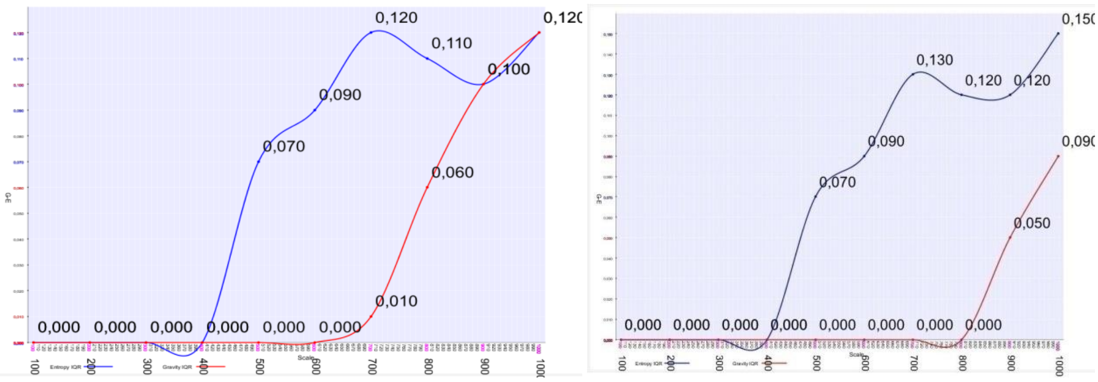

Figure 21. MSA graph for the hypothetical cases of C1 and C2

Table 6. MSA Data Table for the hypothetical case C1

Grid Scale Level	PA (Pixel Area)	Total Cell	NA Cell (exempted)	%	G-IQR	G-Q1	G-Q3	H-IQR	H-Q1	H-Q3
100	15px.15px	4761	272	5.7	0	0	0	0	0	0
200	30px.30px	1225	136	11.1	0	0	0	0	0	0
300	45px.45px	529	88	16.6	0	0	0	0	0	0
400	60px.60px	324	68	20.9	0	0	0	0	0	0
500	75px.75px	196	52	26.5	0	0	0	0.07	0	0.07
600	90px.90px	144	44	30.5	0	0	0	0.09	0	0.085
700	105px.105px	100	36	36.0	0.01	0	0	0.12	0	0.12
800	120px.120px	81	32	39.5	0.06	0	0	0.11	0	0.11
900	135px.135px	64	28	43.7	0.1	0	0	0.1	0	0.1
1000	150px.150px	49	24	48,9	0.12	0	0	0.12	0	0.12

Table 7. MSA Data Table for the hypothetical case C2

Grid Scale Level	PA (Pixel Area)	Total Cell	NA Cell (exempted)	%	G-IQR	G-Q1	G-Q3	H-IQR	H-Q1	H-Q3
100	15px.15px	4761	272	5.7	0	0	0	0	0	0
200	30px.30px	1225	136	11.1	0	0	0	0	0	0
300	45px.45px	529	88	16.6	0	0	0	0	0	0
400	60px.60px	324	68	20.9	0	0	0	0	0	0
500	75px.75px	196	52	26.5	0	0	0	0.07	0	0.065
600	90px.90px	144	44	30.5	0	0	0	0.09	0	0.085
700	105px.105px	100	36	36.0	0	0	0	0.13	0	0.13
800	120px.120px	81	32	39.5	0	0	0	0.12	0	0.12
900	135px.135px	64	28	43.7	0.05	0	0.05	0.12	0	0.12
1000	150px.150px	49	24	48,9	0.09	0	0.09	0.15	0	0.145

The hypothetical case C1 is different from the case B3 has less number of interacting edges, in other words, a smaller number of scale jumps, for all of the eight spatial entities from 32 to 20 in total. As a result, the built system's overall potential for interaction decreases. This brings a smaller number of scale-jumps and less different type of morphologic-interplay. The lower the probability, the lower the uncertainty. Compared to Case B3, the Case C1 is more monotonous with two identical super-blocks in upper and lower ends. As in Table 6, the cumulative entropy, ΣH-IQR, for the Case C1 is 0,60bit, slightly higher, and implying fewer wholeness, compared to the case B3 with 0,57bit ΣH-IQR. In the EI visuals in Figure 19, it is possible to see the most deformed units are those that are overlaid upon the built formations with most scale jumps.

The C2 case is formed through eight equivalent units grouped in two identical and symmetrical modules and centrally situated in equivalent distance, symmetrical location, and position to each other. This hypothetical case study is important about its potential to expose how two non-interacting identical built entities behave in generating the G and H quartiles. This case study, looking at the H and G-IQR columns of the MSA Data Table 7, indicates that the generation of H values is a relational process and the adjacent units' built probabilities affect the H values. See the G-Q3 values for 500-600-700 and 800 grid-scale levels where there are H-Q3 values. This finding proves that the scholars' suggestion for an adjacency factor is a valid and essential need to investigate the relative configuration of built blocks in search of the level of uncertainty gets revealed through such relationality. The way it is established in the context of the proposed method in this study is mostly verified in this hypothetical case. The cumulative H-IQR value for the Case C2 is 0,67bit slightly higher than C1 that implies a bigger potential towards spatial uncertainty.

5. Results and Conclusion

To conclude; Cities changes constantly in different size and portions and in different rhythms. This change inevitably affects the scaling hierarchy of cities what makes everything work in balance. Conventional methods in GIS help to measure and explains spatial change via metric measurements of geographic causalities and geo-located mappings. Proposed method in this paper introduces an analytical approach that measures space on a relational basis across varying scales and generates a value of wholeness.

In Alexander's texts, the concept of wholeness and the life for a spatial setting is a broad notion, and the connection between two concepts is controversial in academia. Building a solid and conclusive analogy between wholeness and life is not always as clear as scholars claim in different texts. However, the entire question of wholeness is a large, flexible and not so clear phenomenon and there is a loose relationship between wholeness and life (Ekinoglu & Kubat, 2017). In another word, beyond strict definitions of "dead" and "alive" life in the space can exist in various degrees in-between. Nevertheless, it is hard to construct a direct and determinant relationship since different levels of life can exist in space with various degrees of wholeness. To avoid this ambiguity, in this study, the concept of wholeness that Alexander depicts is being referred as a quality of "completeness" – an inverse quality of uncertainty - that emerges through the relationships among the sub-constituents & the whole system relationships across scales. (Ekinoglu *et al*., 2017).

Three groups of hypothetical case studies with slight and excessive differences demonstrate meaningful relationships between the scaling characteristics of the spatial layouts and the entropy values created. The critical finding is that the quartiles allow controlling the extremes of the scalar data and the user of the analysis tool can be specific about the built subunits that disrupt and undermine the scaling hierarchy and lead to high entropy for that particular scale level. This will minimize the loss of scaling signature and the harming effects of new developments in existing urban areas.

For practitioners and local governments, the proposed method is allowing to run evidence-based measuring of urban change scenarios in the built environment and testing the after-effects of design and planning scenarios before the field work. In doing so, local governments and policymakers can monitor environmental change in the development of urban areas for human-wellbeing.

Results derived from the case studies reveal and supported by the below findings;

1- Not only the way the spatial entities get clustered but also their geographic distributions entropically matters. This outcome is supported by Leibovici's (2009) suggestion about generating entropy for spatial distributions requires considering the proximities as an adjacency factor. Such kind of adjacency system is a valid need to investigate the relative configuration of built blocks in search of the level of uncertainty, entropy, revealed through such relationality.

2- The higher possibility of solid-void interactions, the more volatile G (unit-specific-built-probability) and thus H (entropy of a max adjacent unit) states. This finding proves that entropic state, from the morphologic layout point of view, is a scale-dependent quality, in other words, the scale of the grid system that frames the morphologic relations through equivalent units is a major determinant factor.

3- The less and balanced scale-jumps in the analyzed spatial setting, the less different type of morphologic-interplay. This brings a less volatile G (unit-specific-built-probability) and H (entropy of a max adjacent unit) values, if not zero or close to zero, in a stable way.

4- As a common finding in all the hypothetical case studies, built portions that fall into grid units start to affect the Q1 and Q3 in larger scale levels where the quantity of the grid units start to decrease –larger scales generate larger but less amount of grid units- and the built portions start to fall into the units more than before and affect the quartiles by the end of the first 25% (Q1) and by the beginning of the last 25% (Q3) of the data in interquartile range statistics method's terms.

5- Looking at the MSA graphs, one can say that there is an obvious parallel movement in up and downshifts of the G-IQRs and H-IQRs on the MSA graphs and this indicates that changing grid scales eventually reproduce G datasets, which in return reproduce H datasets. This happens because there is a scale-led co-dependency between each other.

6- This implies that even with the missing built pieces that are identical in quantity and structural & geometric features, their geographic positions do matter in the generation of entropy for a particular area.

7- Somewhat identically grouped clusters remind a definitive torn or an in-between border that disconnects every possible spatial interaction while it also enables a wide range of different potential scenarios. Such a situation, by all means, creates uncertainty about the design process and more specifically about possible spatial interventions. By definition of information entropy, the higher the number of data types in a particular information, the lower the probability of the same type of data to get together. In spatial terms, the higher morphologic ability in a space, the more uncertainty it triggers in design thinking.

8- In a monotonous context of identical massive sub-constituents, the system reveals approximate entropic states (H-IQR) in each scale level due to recurring scaling hierarchy and repeating fractal geometry in every scale level. One can conclude that such kind of order implies stable or repeatedly approximate entropic situations since the unit-specific G values stay same or approximate in varying scale levels. Whether or not such kind of entropic stability is a positive spatial quality is the topic of another study since massive multi-scalar order does not guarantee wholeness. Alexander (2002-2005) himself also confirms this remark in the "*Nature of Order*".

9- Briefly, the method is set upon a logic of bilateral flow of spatial push and pull factors based on scale and distance. Entropic Interaction way of spatial modeling in this sense allows designers to see spatial effects of massive and volumetric relations through a deformed grid and be proactive against scale jumps that might harm the scaling hierarchy in the built environment in several topics from energy to socio-economic issues.

Acknowledgment. This research has been granted by TUBITAK during Dr. Ekinoglu's residency as a visiting scholar at Columbia University, Earth Institute, Urban Design Lab. He would like to thank R. Plunz, A.S.Kubat, Z. Ulusoy, N. Ergun for their feedbacks and Si Ying Du and Bora Koç for their great efforts and contributions in the development of the tool and also to Prof. Nikos Salingaros for his invaluable feedback and contribution

References

Alexander, C. (2002-2005). *The nature of order: an essay on the art of building and the nature of the universe (4-Books).* Berkeley, California: Center for Environmental Structure.

Bailey, K.D. (2015, May). *Entropy systems theory.* Retrieved from Encyclopedia of Life Support Systems (EOLSS) Retrieved from http://www.eolss.net/ebooks/Sample%20Chapters/C02/E6-46-01-04.pdf

Brannon, R. (2008). *Kinematics: The mathematics of deformation.* Retrieved from http://www.mech.utah.edu/:

http://www.mech.utah.edu/~brannon/public/Deformation.pdf

Crooks, A., Castle, C. & Batty, M. (2007). *Key Challenges in Agent-Based Modelling for Geo-Spatial Simulation.* Centre for Advanced Spatial Analysis (CASA) Working Paper.

Ekinoglu, H., Kubat, A.S. (2017). Analytical measuring and visualization of spatio-temporal change in built environments: Beyazit Square case study area in Istanbul. *Journal of Planning*, *27*(3), 362-374.

Ekinoglu, H., Kubat A.S., Plunz, R. (2017). Modeling spatial wholeness in cities using information entropy theory A|Z *ITU Journal of the Faculty of Architecture*, 14(3), 47-61.

Gatrell, A.C. (1983) Distance and Space: A Geographical Perspective Contemporary problems in geography, Clarendon Press.

Jat, M.K., Garg, P.K., & Khare, D. (2007). Monitoring and modeling of urban sprawl using remote sensing and GIS techniques. *International Journal of Applied Earth Observation and Geoinformation*, *10*(1), 26-43.

Jiang, B. (2017). A topological representation for taking cities as a coherent whole. *Geographical Analysis*. 50(3). https://doi.org/10.1111/gean.12145

Leibovici, G.D. (2009). *Defining spatial entropy from multivariate distributions of co-occurrences.* In K. S. Hornsby, & e. a. (eds), Spatial Information Theory (pp. 392-404). amber W'rach: Springer-Verlag Berlin Heidelberg.

Miller, H. (2004). Tobler's first law and spatial analysis. *Annals of the Association of American Geographers*, 284-289. Retrieved from: http://dds.cepal.org/infancia/guia-para-estimar-la-pobreza-infantil/bibliografia/

Miller, H.J.; Wentz, E.A. (2003) Representation and spatial analysis in geographic information systems. *Annals of the Association of American Geographers*, *93*(3), 574-594.

Macal, C., & North, M. (2005). Tutorial on Agent-Based Modeling and Simulation. Winter Simulation Conference. Orlando, Florida.

Salingaros, N. (2018). Organized Complexity vs. Random Adaptation. *New Design Ideas*, *2*(2), 5-19.

Salingaros, N., West, B.J. (2005). *A Universal Rule for the Distribution of Sizes*, Environment and Planning B 26 (1999), pages 909-923. Chapter 3 of *Principles of Urban Structure*, Techne Press, Amsterdam, Holland, 2005.

Salingaros, N.A. (1997). Life and Complexity in Architecture From a Thermodynamic Analogy, *Physics Essays*, 10, 165-173.

Shannon, C.E. (1948). A mathematical theory of communication. *Bell System Technical Journal*, *27*(3), 379-423.

Tobler, W. (1970). A computer movie simulating urban growth in the Detroit region. *Economic Geography*, *46(2)*, 234-240.

Turner, A. (2001). Depthmap: *A program to perform visibility graph analysis.* 3rd International Symposium on Space Syntax. Atlanta: Georgia Institute of Technology.

Turner, A., & Penn, A. (1999). *Making isovists syntactic: Isovist integration analysis.* Proceedings of the 2nd International Symposium on Space Syntax Vol. 3. Brasilia, Brazil: Universidad de Brasil. http://www.sciencedirect.com/science/article/pii/S0303243407000335

Versoza, L.C., & Gonzales, R.M. (2010). Remote sensing, geographic information systems, and Shannon's entropy: Measuring Urban Sprawl in A Mountainous Environment. ISPRS TC VII Symposium - 100 Years, (pp. 269-274). Vienna.

Wang, T. (2016). *Information & Entropy (Comp 595 DM).* Retrieved from http://www.csun.edu/~twang/595DM/Slides/Information%20&%20Entropy.pdf

Yuksel, C., Schaefer, S., Keyser, J. (2011). Parameterization and applications of Catmull–Rom curves, *Computer-Aided Design*, *43*(7), 747-755.